A New Decomposition of $\pi\pi$ S-wave Interaction *

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Abstract

The experimental isoscalar $\pi\pi \to \pi\pi$ S-wave amplitude squared below 1.75 GeV is characterized by a very broad structure f_0(400-1200) with two narrow dips due to its destructive interference with the $f_0(980)$ and $f_0(1500)$. The $f_0(1370)$ and $f_0(1710)$ do not show up clearly in the $\pi\pi \to \pi\pi$ S-wave amplitude due to their weak coupling to $\pi\pi$. This paper is about the controversial nature of the broad f_0(400-1200). We decompose it into two parts, *i.e.*, t-channel ρ meson exchange plus an additional s-channel resonance $f_0(X)$. With the t-channel ρ meson exchange fixed by the isotensor $\pi\pi \to \pi\pi$ S-wave scattering, we re-fit the CERN-Munich (CM) data on $\pi\pi$ scattering to get parameters for the $f_0(X)$. We find that the $f_0(X)$ is very broad with a nearby pole at $(1.67 - 0.26i)$ GeV, while the t-channel ρ meson exchange part produces a pole at $(0.36 - 0.53i)$ GeV. The implication of our results for the study of σ and glueball is discussed.

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1 Introduction

As stated by the Particle Data Group[1], the $I = 0$ $J^{PC} = 0^{++}$ sector is the most complex one, both experimentally and theoretically, in the light quark meson spectroscopy. Meanwhile it is also the most important and intriguing sector. It has the quantum number of the vacuum, the σ meson and the lightest glueball. The importance of the σ in relation to the dynamical chiral symmetry breaking[2] and in reproducing the strong interaction between nucleons[3] has long been known. The lightest glueball are predicted to be around $1.5GeV$ by various lattice QCD calculations[4] and all sorts of QCD inspired models[5]. However the identification of both σ and glueball is still not settled.

A good place to study the properties of these isoscalar 0^{++} particles is the $\pi\pi$ S-wave scattering process, since if a 0^{++} resonance has a substantial coupling to $\pi\pi$, then it should show up clearly in the $\pi\pi \rightarrow \pi\pi$ S-wave amplitude. In this context, the $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering from the old πN scattering experiments with both unpolarized[6] and polarized targets[7] has been re-analyzed[8, 9] in combination with new information from $p\bar{p}$ and other experiments. The $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering has also been studied by E852[10] and GAMS[11] Collaborations recently. All these efforts result in a consistent picture for the isoscalar $\pi\pi$ S-wave intensity, i.e., a broad structure f_0(400-1200) with two narrow dips due to its destructive interference with the $f_0(980)$ and $f_0(1500)$. The $f_0(1370)$ and $f_0(1710)$ do not show up clearly in the $\pi\pi \rightarrow \pi\pi$ S-wave amplitude due to their weak coupling to $\pi\pi$[8, 12].

Although the existence of the broad f_0(400-1200) resonance is experimentally well established[1], its nature is still far from settled. Some people believe that it is an intrinsic pole due to $q\bar{q}$ σ meson[13, 14] or glueball[15, 16] while others claim that it is a dynamical pole mainly due to t-channel exchange forces[17, 18, 19, 20, 21]. A criticism[22] on the intrinsic pole approaches is their neglect of the“exotic” isotensor $\pi\pi$ scattering channel. In the dynamical pole approaches, the isotensor $\pi\pi$ scattering channel can be used to set the scale of t-channel forces.

However, in the t-channel exchange approaches[17, 18] with constraints from the isotensor $\pi\pi$ scattering channel, the t-channel exchanges alone under-estimate the broad structure in the isoscalar $\pi\pi$ scattering channel. Some addition S-channel resonance contribution is needed to full reproduce the broad structure. Refs.[17, 18] attribute it to the tails of higher resonances

such as $f_0(1370)$ and $f_0(1500)$, while others[13, 23] believe that an intrinsic $q\bar{q}$ σ is necessary.

From information of other sources[12, 24], we know that the $f_0(1370)$ couples very weakly to $\pi\pi$ and the $f_0(1500)$ is rather narrow. Both are unlikely to compensate the discrepancy between the t-channel exchange calculations[17, 18] and the data. The t-channel exchange calculation of Ref.[19] without any free parameter and form factors reproduce the isoscalar $\pi\pi$ S-wave phase shifts very well, but over-estimate the isotensor $\pi\pi$ S-wave phase shifts[20]. The off-shell form factors for the t-channel exchange mesons is necessary to reproduce the isotensor $\pi\pi$ S-wave data[20] and will result in an under-estimation of isoscalar $\pi\pi$ S-wave phase shifts as in Refs.[17, 18]. Some additional s-channel resonance seems necessary.

In this paper, we decompose the broad f_0(400-1200) structure into two parts, *i.e.*, t-channel ρ meson exchange plus an additional s-channel resonance $f_0(X)$. With the t-channel ρ meson exchange form factor parameter fixed by the isotensor $\pi\pi \to \pi\pi$ S-wave scattering, we re-fit the old CERN-Munich (CM) data[6] on $\pi\pi$ scattering to get parameters for the $f_0(X)$.

2 Formalism

2.1 t-channel ρ meson exchange amplitude

We follow the K-matrix formalism as in Refs.[19, 20]. In order to explain the $\pi\pi$ $I = 2$ S-wave experimental data, a form factor is needed to take into account the off-shell behavior of the exchanged mesons. We use a form factor of conventional monopole type at each vertex:

$$F(q^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 + q^2}, \tag{1}$$

where m and q is the mass and four-vector momentum of exchanged mesons, and Λ is the cutoff parameter which will be determined by experimental data. Then the Born term for the ρ exchange is:

$$T^{Born}(I=0) \quad = \quad 2G[(\frac{\Lambda^2 - m_\rho^2}{\Lambda^2 - t})^2 \frac{s-u}{m_\rho^2 - t} + (\frac{\Lambda^2 - m_\rho^2}{\Lambda^2 - u})^2 \frac{s-t}{m_\rho^2 - u}], \tag{2}$$

$$T^{Born}(I=2) \;\; = \;\; -\frac{1}{2}T^{Born}(I=0) \tag{3}$$

where s, t, u are the usual Mandelstam variables and $G = g^2_{\rho\pi\pi}/32\pi$=0.364. Their S-wave projections are

$$K_S^{I=0}(s) \;\; = \;\; 4G\{(\frac{m_\rho^2}{\Lambda^2}-1)\frac{\Lambda^2+2s-4m_\pi^2}{\Lambda^2+s-4m_\pi^2}+\frac{2s+m_\rho^2-4m_\pi^2}{s-4m_\pi^2}ln\frac{(s+m_\rho^2-4m_\pi^2)\Lambda^2}{(s+\Lambda^2-4m_\pi^2)m_\rho^2}\}, \tag{4}$$

$$K_S^{I=2}(s) \;\; = \;\; -\frac{1}{2}K_S^{I=0}(s). \tag{5}$$

K-matrix unitarization is introduced by

$$T_S^I(s) = \frac{K_S^I(s)}{1-i\rho_1(s)K_S^I(s)}, \tag{6}$$

where $\rho_1(s) = (1-4m_\pi^2/s)^{1/2}$ is the phase space factor. The relation between the S-wave amplitude and the phase shift parameters δ_I and η_I is

$$T^I(s) = \frac{\eta_I(s)e^{2i\delta_I(s)}-1}{2i\rho_1(s)}. \tag{7}$$

From above formalism, we get the $\pi\pi$ $I = 2$ S-wave phase shift $\delta_2(s)$ as shown in Fig.1(a) for a few choices of Λ parameter. The dot-dashed curve without introducing the form factor obviously overestimate the isotensor $\pi\pi$ S-wave phase shifts. As in Ref.[20], the solid line with Λ = 1500 MeV reproduces the experimental isotensor $\pi\pi$ S-wave phase shifts best. We also investigated the contribution from t-channel $f_2(1275)$ exchange. It gives marginal improvement for the fit to the data with a form factor cutoff parameter around 1 GeV as in Ref.[18]. Hence we do not include this contribution in our further calculations.

Fig.1(b) shows the corresponding contribution of ρ exchange to the $I = 0$ S-wave phase shift. Including the form factor significantly reduces the t-channel ρ exchange contribution. The t-channel ρ meson exchange amplitude has a pole at $(0.36 - 0.53i)$ GeV.

With the t-channel ρ meson exchange fixed by the isotensor $\pi\pi \rightarrow \pi\pi$ S-wave scattering, we will re-fit the CERN-Munich data to get information for s-channel resonances.

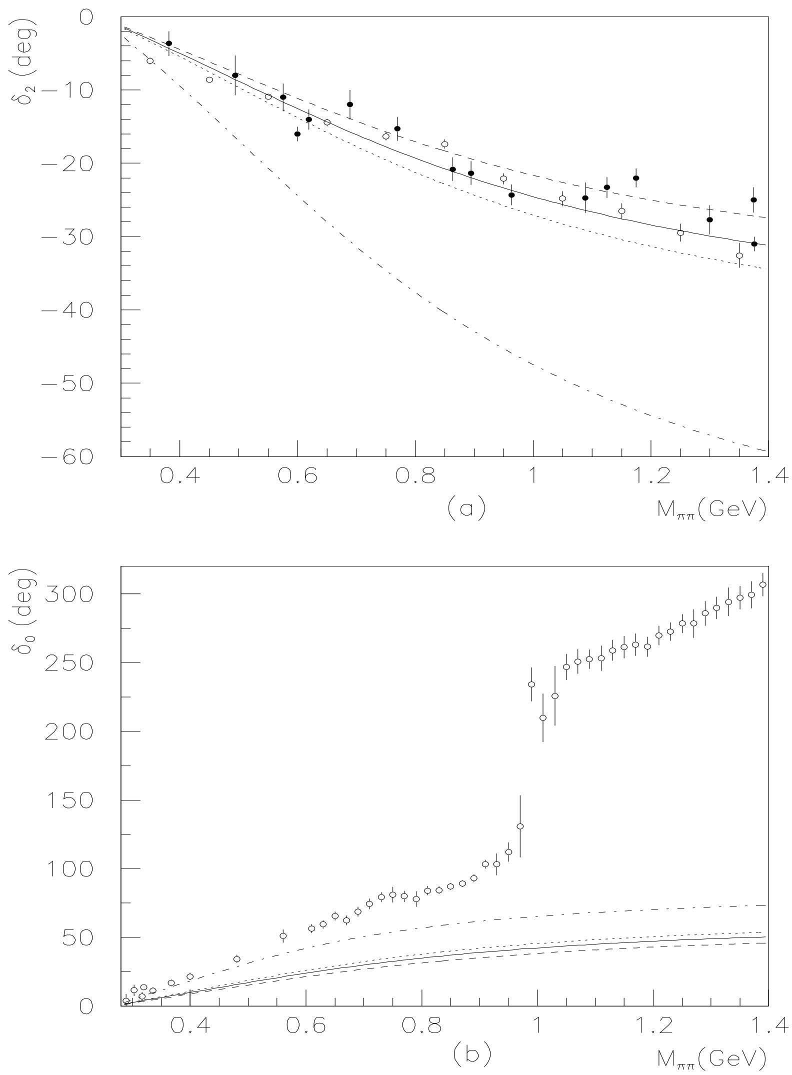

Figure 1: The $I = 2, 0$ S-wave phase shift δ_2 (a) and δ_0 (b) for $\pi\pi$ scattering. The experimental data for δ_2 are from Ref.[25] (circles) and Ref.[26](dots); The data for δ_0 are from Ref.[6, 27]. The theoretical curves are from our t-channel ρ exchange calculations with $\Lambda = 1400$ MeV (dotted lines), $\Lambda = 1500$ MeV (solid lines), $\Lambda = 1600$ MeV (dashed lines) and without form factors (dot-dashed lines).

2.2 Full partial-wave amplitudes

For each partial wave, it usually contains more than one components. Specifically for isoscalar $\pi\pi$ S-wave, besides the t-channel ρ exchange contribution, there are also contributions from several s-channel resonances. To combine several components in a single partial wave, we use the Dalitz-Tuan prescription[8]. Suppose two components a and b for the partial wave l are expressed individually as

$$T_l^a(s) = \frac{G_a}{C_a(s) - iG_a\rho_1(s)} \text{ and } T_l^b(s) = \frac{G_b}{C_b(s) - iG_b\rho_1(s)},$$

then the combined amplitude will be

$$\hat{T}_l^{ab}(s) = \frac{G_aC_b(s) + G_bC_a(s)}{[C_a(s) - iG_a\rho_1(s)][C_b(s) - iG_b\rho_1(s)]} \tag{8}$$

$$= \frac{G_{ab}}{C_{ab}(s) - iG_{ab}\rho_1(s)}, \tag{9}$$

with $C_{ab}(s) = C_a(s)C_b(s) - G_aG_b\rho_1^2(s)$, and $G_{ab} = G_aC_b(s) + G_bC_a(s)$. At either component a simple pole term survives. The amplitude is explicitly unitary, and the denominator contains poles from both components. Further poles are added one by one using the equations above.

In fitting CERN-Münich data, we use the same formulae as the approach A in Ref.[8] except for $\pi\pi$ S-waves. For the isotensor $\pi\pi$ S-wave, Ref.[8] used a scattering length formula, here we use the t-channel ρ exchange amplitude; for the isoscalar $\pi\pi$ S-wave, Ref.[8] used an effective S-channel σ resonance to describe the f_0(400-1200) broad structure, here we decompose it into two components, *i.e.*, t-channel ρ exchange plus an S-channel resonance $f_0(X)$.

3 Numerical results and discussion

With the t-channel ρ exchange parameter $\Lambda = 1.5GeV$ fixed by the isotensor $\pi\pi$ phase shifts, we re-fit the CM data on the moments of the $\pi\pi$ decay angular distributions. The parameters for relevant resonances are fixed to the values of Ref.[8] except for $f_0(X)$ and $f_0(980)$ which are responsible for the low energy isoscalar $\pi\pi$ S-wave together with the t-channel ρ exchange. Among the moments of the CM data, the $N < Y_1^0 >$ and $N < Y_2^0 >$[8]

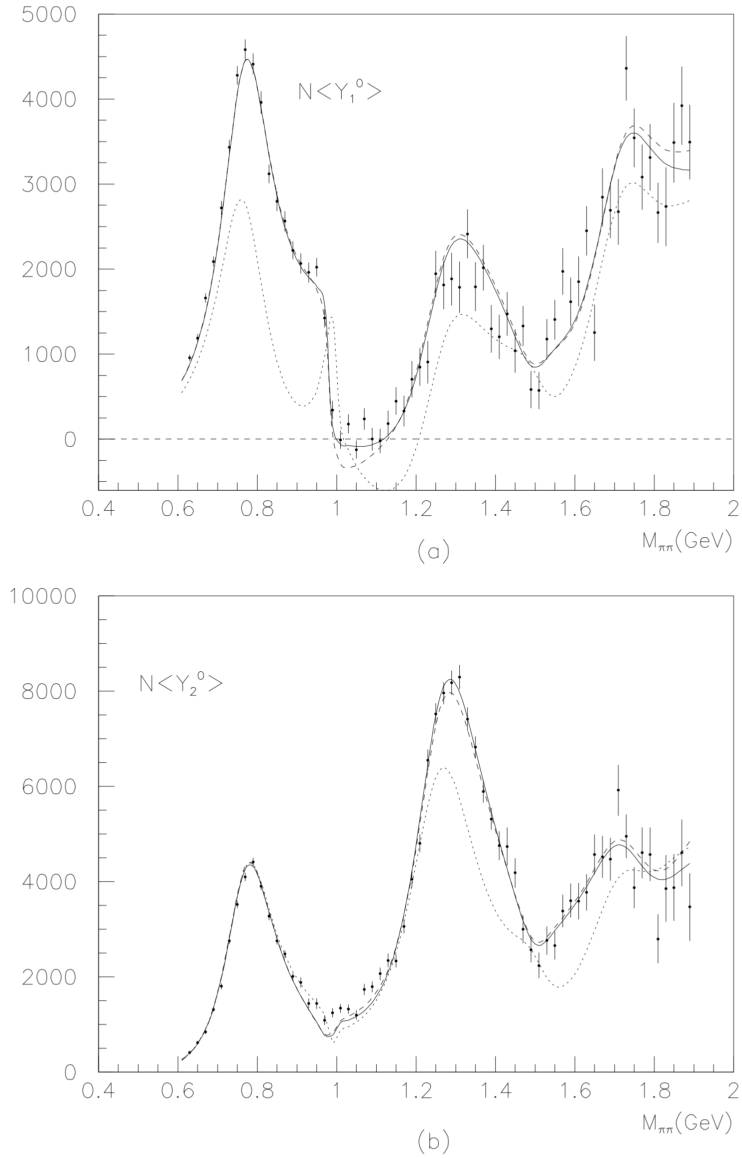

Figure 2: The fits to CM data, the dashed line is the previous solution of Ref.[8], the solid line is our fit and the dotted line is for neglecting the contribution of $f_0(1670)$.

are the most sensitive ones to the isoscalar $\pi\pi$ S-wave. Fig. 2 shows the CM data, the previous solution of Ref.[8] (dashed lines) and our fit (solid lines) to these moments. The fitted parameters for the $f_0(X)$ are $M_R = 1.71$ GeV, $\Gamma_{\pi\pi} = 1.12$ GeV and $\Gamma_{4\pi} = 0.16$ GeV which simulates all inelastic channels as in Ref.[8] for the effective broad σ. This $f_0(X)$ has a nearby pole at $(1.67 - 0.26i)$ GeV, so we denote it as $f_0(1670)$. As a comparison, we also show the results without the $f_0(1670)$ by the dashed lines in Fig. 2. It obviously fails to reproduce data. The fitted parameters for the $f_0(980)$ are $M_R = 0.982$ GeV, $g_{\pi\pi} = 0.146$ GeV and $g_{K\bar{K}} = 0.562$ GeV in its Flatte formula $1/(M_R^2 - s - ig_{\pi\pi}\rho_{\pi\pi} - ig_{K\bar{K}}\rho_{K\bar{K}})$. The corresponding full isoscalar $\pi\pi$ S-wave amplitude squared is shown in Fig.3. It is characterized by a very broad structure with two narrow dips due to its destructive interference with the $f_0(980)$ and $f_0(1500)$. The other two PDG established 0^{++} resonances, $f_0(1370)$ and $f_0(1710)$, do not show up clearly in the $\pi\pi \to \pi\pi$ S-wave amplitude due to their weak coupling to $\pi\pi$[12].

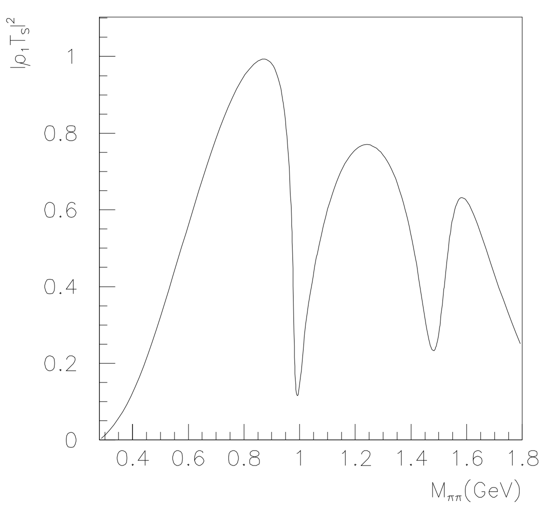

Figure 3: The $I = 0$ $\pi\pi$ S-wave amplitude squared.

In summary, besides the $\sigma(400)$ pole at $(0.36 - 0.53i)$ GeV produced by

the t-channel ρ exchange, there are five s-channel resonances below 1.75 GeV: four relative narrow PDG established ones, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, plus a very broad one found here, $f_0(1670)$ with a nearby pole at $(1.67 - 0.26i)$ GeV. Theoretically there could be two resonances from 1^3P_0 $q\bar{q}$ nonet, two from 2^3P_0 nonet, then an extra from the lightest glueball[15]. The bare glueball state is dispersed over three real resonances, $f_0(1500)$, $f_0(1710)$ and $f_0(1670)$. This may give a nature explanation that $f_0(1500)$ and $f_0(1710)$ are produced very strongly in the glue-rich process J/Ψ radiative decays[1, 28]. The $f_0(1670)$ may not be easy to be identified in production processes due to its large decay width.

Note that glueballs could be very broad[29]. Lattice QCD[4] predicts the $\pi\pi$ decay width alone of the lightest glueball to be (108 ± 29) MeV. Considering its possible larger decay width to $\sigma\sigma$, $\rho\rho$ etc., the bare glueball state could already have very large decay width. The mixing of overlapping resonances could further increase the width of one state and simultaneously reduce the width of another one[15].

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